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LETTERS TO THE EDITOR

The recent paper by Gresho *et al.*[1] raises issues regarding the stability of two-dimensional flow over a backward-facing step, and related issues regarding the accuracy requirements of CFD. Good practitioners of CFD require an understanding of numerical analysis, practical scientific computing, and the physics of the flow they are calculating. Lack of attention to any of these aspects of CFD can lead to misleading or incorrect results. Indeed, the stability of the flow over a backward-facing step presents an excellent prototype of the kinds of pitfalls one may encounter. In particular, the paper by Gresho *et al.*[1] correctly concludes that the step flow is 'stable' and steady at $Re = 800$ provided that a certain formally correct but physically unrealistic notion of stability is applied.

It was stated in the Gresho *et al.*[1] paper that their work was motivated by the results contained in our paper.[2] Our paper addressed the secondary three-dimensional instability of flow over a backward facing step, assuming the existence of a primary two-dimensional instability. Gresho *et al.*[1] have pointed out that the backward-facing step flow with expansion ratio $r = 1:2$ is absolutely stable for Reynolds numbers $Re = 800$ (and higher); here the Reynolds number is based on $2 \times Q/\nu$, where Q is the flow rate and ν is the kinematic viscosity. Nevertheless, an 'instability' was reported in our paper on the basis of direct numerical simulations of the flow. At first glance, it would appear that there is either an inconsistency or an error. However, our results and those of Gresho *et al.*[1] are, in fact, consistent; the flow is, in a formal sense, absolutely stable at $Re = 800$ but, in a practical sense, it is unstable and will be observed as such both experimentally and in computer simulations designed to properly mimic the experiments. The explanation of the apparent paradox is as follows.

The stability analysis of Gresho *et al.*[1] is performed using temporal stability methods in which an initial non-zero perturbation is imposed on the flow. At $Re = 800$, these perturbations die out, so the flow is temporally (and absolutely) stable. However, in comparing with experiment, it is necessary to perform *a spatial stability* analysis, which is complicated here, of course, by the non-parallel nature of the base flow which exhibits separation around the bottom of the step. For slowly varying mean profiles, like those exhibited by typical boundary layers, Gaster's transformation[3] (based on the analyticity of the eigenvalue dispersion relation) shows that there is a close relation between temporal and spatial modes. In the case of backward-facing step flow, this close relationship breaks down so that the temporal stability analysis gives but little insight into the true nature of the stability of the flow. Indeed, while the flow is absolutely (temporally) stable, it is convectively (spatially) unstable for $Re \gtrsim 700$. This convective instability will control the physically observed flow unless disturbances can be reduced to an exponentially small level, which is unrealistic physically. For this reason, the temporal stability analysis and the stability conclusions of Gresho *et al.*[1] do not relate to physically realizable step flows.

Convective instability for $Re \gtrsim 700$, means that external disturbances can be spatially amplified. A disturbance at the upstream boundary that is localized in time will propagate as a localized spatial disturbance with increasing amplitude as it convects downstream. At very large distances downstream, sufficiently small amplitude disturbances at inflow will die out because the mean flow becomes parabolic Poiseuille channel flow (since plane Poiseuille flow is stable for $Re \lesssim 5772$) and parabolic channel flow is then spatially stable. However, the region of convective growth is quite long for $Re \gtrsim 700$ and the maximum amplification is large; at $Re = 700$ ($Re = 1000$), the spatial growth is one (two) orders of magnitude over a distance of 25 step heights. The region of spatial convective growth increases approximately linearly with Re so the cumulative growth increases at least exponentially with Re. Furthermore, sustained external excitations at moderately small amplitudes will typically lead to a time-periodic asymptotic state. This is demonstrated through different types of excitation including random noise as an inflow condition.[2,4] These numerical simulations show that, above a (small) threshold noise level, an unsteady response corresponding to a finite amplitude oscillation is obtained. These results are consistent with available experimental data and account for the different asymptotic flow states obtained in numerical simulations.

As regards the accuracy of spectral element methods for the calculations at hand, they have proven efficiency and utility compared to the other methods espoused by Gresho *et al.*[1] At a comparable or lower resolution the spectral element method yields the same results as the other methods employed by Gresho *et al.*[1] (In the spectral element calculations reported by Gresho *et al.*[1] the input file is wrong and leads to unacceptable divergence errors as was revealed to us by one of the co-authors (Torczynski)).

In summary, we believe that in addition to the obvious requirement that converged numerical solutions be obtained, it is also necessary to set up the problem properly in order to capture the essential physics. Error estimates and mathematical convergence proofs are properly sought only for properly posed problems. To amplify a quote from the paper of Gresho *et al.*[1] 'CFD is not easy, even today; much care is required' to solve the problem properly! In brief, to compare CFD simulations of the backward-facing step with experiments, the temporal stability analysis of Gresho *et al.*[1] is simply not relevant; convective stability analysis is required.

G. E. Karniadakis and S. A. Orszag

REFERENCES

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AUTHOR'S REPLY

The Letter to the Editor by Karniadakis and Orszag provides a brief synopsis of the ideas associated with convective instabilities and their differences with the more commonly recognized temporal instabilities. Unfortunately, the authors have misinterpreted the point of our paper[1] which has nothing to do with physically realizable flows or convective instabilities. In Reference 2 a two-dimensional, boundary value problem with time-independent boundary conditions (and no external perturbations) was posed for a backstep at $Re=800$. The conclusion from their time-dependent, numerical simulation was that the solution to this problem was time-dependent. This is the same conclusion reported by this same group when they attempted to solve the well-defined problem of the OBC Minisymposium. The paper by Gresho *et al.*[1] was constructed to show that this conclusion by Kaiktsis *et al.*[2] at least for the problem of the Minisymposium, is incorrect and that the solution to the boundary value problem is, in fact, time-independent (steady) and stable to significant external perturbations. The erroneous conclusion by Kaiktsis *et al.*[2] was due to poor mesh refinement. At no time did Gresho *et al.*[1] claim that this was a simulation of a real flow or attempt to compare results with experiments; two-dimensional flows over a step are well known to be realizable only at much lower Reynolds numbers (see Reference 3). The Gresho *et al.*[1] paper is not about the stability of physical flow; rather, it is about an incorrect numerical solution to a boundary value problem. Also, the purported revelation of an incorrect input file attributed to J. R. Torczynski is simply untrue and serves only to distract attention from the fact that the spectral-element method, when properly used, produces results in harmony with other numerical methods.

REFERENCES

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